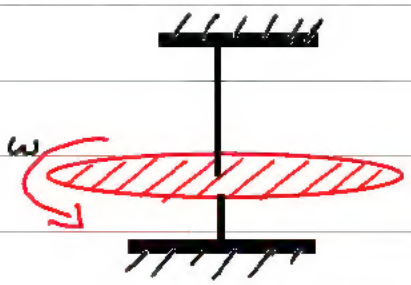
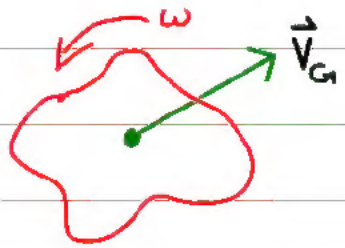
Meriam 6/8: Angular Momentum for 2D bodies

Angular momentum equations developed for particles are applicable to solid bodies:



$$(\vec{H}_o)_2 = (\vec{H}_o)_1 + \sum_{i=1}^{N} \int_{t_1}^{t_2} (\vec{M}_o)_i \, dt$$



$$(\vec{H}_G)_2 = (\vec{H}_G)_1 + \sum_{i=1}^{N} \int_{t_1}^{t_2} (\vec{M}_G)_i \, dt$$

where

$$\int_{t_1}^{t_2} \vec{M}_G \, dt = (\vec{M}_G)_{average} \, \Delta t$$

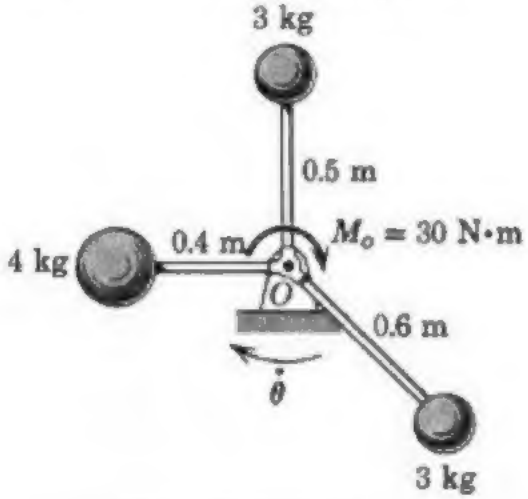
$$\int_{t_1}^{t_2} \vec{M}_o \, dt = (\vec{M}_o)_{average} \, \Delta t$$

} angular impulse

$$H_o = I_o \omega$$

$$H_G = I_G \omega$$

The three small spheres are welded to the light rigid frame, which is rotating in a horizontal plane about a vertical axis through point O with an angular velocity $\dot{\theta} = 20$ rad/s. If a couple $M_O = 30$ N·m is applied to the frame for 5 s, compute the new angular velocity $\dot{\theta}'$.



For entire assembly $\Sigma M_O = \dot{H}_O$, $\int \Sigma M_O \, dt = \Delta H_O$

$$\int \Sigma M_O \, dt = M_O t = \Delta\left[\Sigma m_i r_i (r_i \dot{\theta})\right] = \Sigma m_i r_i^2 \Delta\dot{\theta}$$

$$30(5) = [3(0.5)^2 + 4(0.4)^2 + 3(0.6)^2](\dot{\theta}' - 20)$$

$M_O t = I_O \Delta\omega$

$150 = 2.47(\dot{\theta}' - 20)$, $\boxed{\dot{\theta}' = 80.7 \text{ rad/s}}$

JLM

$$\int \Sigma M_O \, dt = \Delta H_O = (H_O)_2 - (H_O)_1$$

$$= \sum_{i=1}^{3} (\Delta H_O)_i = \sum_{i=1}^{3} \Delta(m_i r_i v_i)$$

$$= \sum_{i=1}^{3} \Delta(m_i r_i [r_i \dot{\theta}]) = \sum_{i=1}^{3} m_i r_i^2 \Delta\dot{\theta}$$

$$M_O t = \Delta\dot{\theta} \sum_{i=1}^{3} m_i r_i^2$$

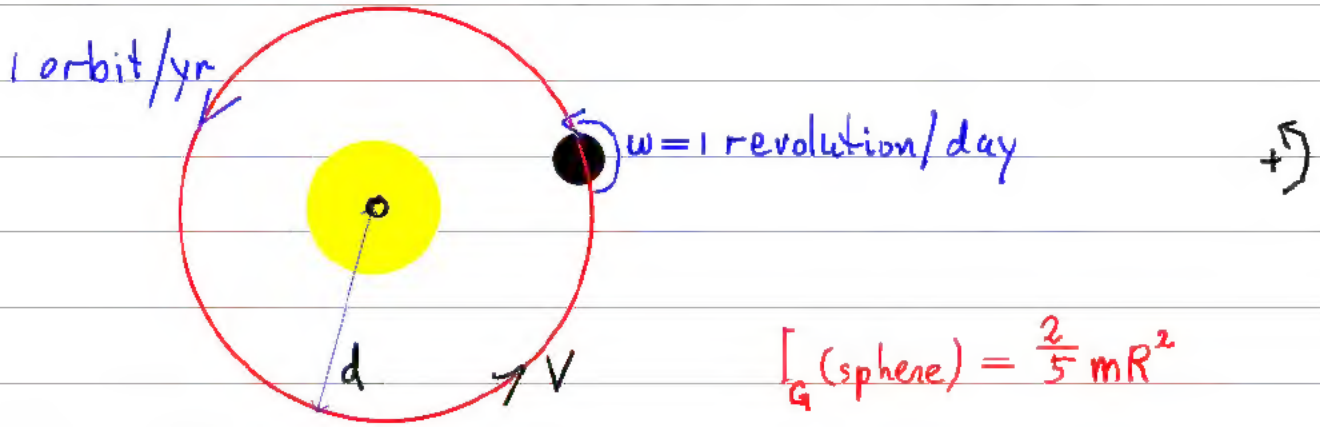
$$= (\Delta\omega)(I_O)$$

$$M_O t = (I_O)(\dot{\theta}' - 20 \text{ s}^{-1})$$

Find the angular momentum of a rotating body that is orbiting about a point O.

Eg. Find $\vec{H}_o$ of planet earth about the sun.

Looking down on the earth from above the north pole,



I_G (sphere) $= \frac{2}{5} mR^2$

$$\vec{H}_o = I_G \omega + mvd \quad \curvearrowleft$$

$$= \left(\frac{2}{5} m_e R_e^2\right)\left(1 \text{ rev/day} \times \frac{1 \text{ day}}{86400 \text{ s}} \times \frac{2\pi \text{ radians}}{\text{rev}}\right) +$$

$$(m_e)\left[(2\pi \times \text{orbital radius})/(86{,}400 \times 365 \text{ s})\right](\text{orbital radius})$$

where $m_e = 6E24$ kg

$R_e = 6.4E6$ m

d = orbital radius $\cong 1.5E11$ m

$$\vec{H}_o = 7.1E33 + 2.7E40$$
$$= 2.7E40 \text{ kg} \cdot \text{m}^2/\text{s}$$

E.g.

Given:

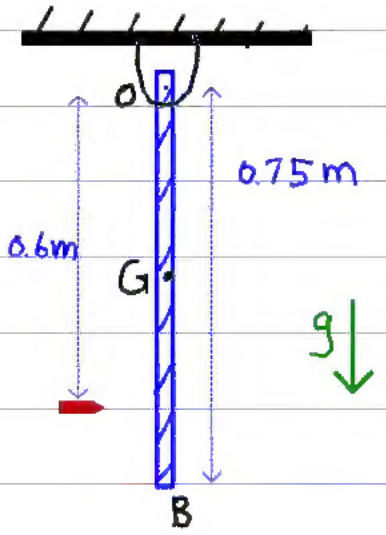
$m_{AB} = 4\,kg$

$m_{bullet} = 30\ grams$

$v_{bullet} = 400\ m/s$

bullet imbeds in OB

initial ω of OB $= 0$

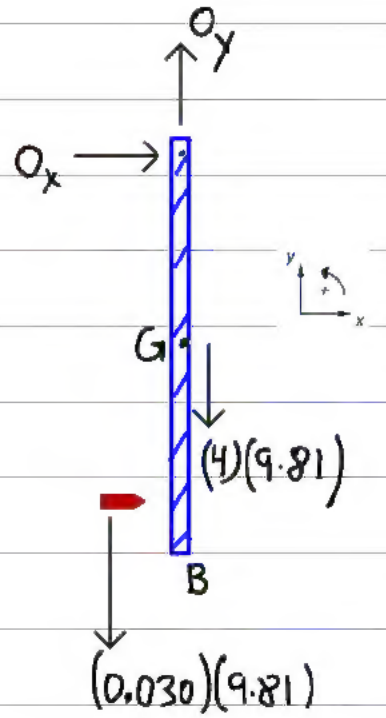


Find $\vec{v}_G$ immediately after impact

Draw FBD

Select coordinate system

Consider very quick trip:
from just before impact (1)
to just after impact (2)



Possible routes:

method 1/ $\vec{a} = \vec{F}/m \rightarrow \vec{v}_2 = \vec{v}_1 + \int_{t_1}^{t_2} \vec{a}\, dt$

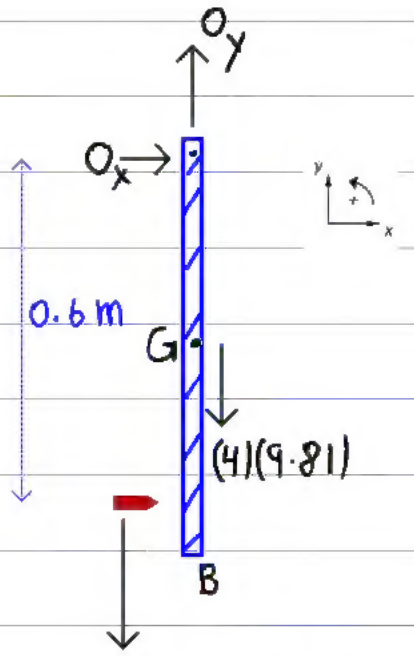
method 2/ $T_2 = T_1 + U_{1\rightarrow 2}$ – energy converted to heat & deformation

method 3/ $(G_x)_2 = (G_x)_1 + \int_{t_1}^{t_2} O_x\, dt$

method 4/ $(\vec{H}_o)_2 = (\vec{H}_o)_1 + \int_{t_1}^{t_2} \vec{M}_o\, dt$ ★

For the bullet / bar system collision:

$$(H_O)_2 = (H_O)_1 + \int M_O\, dt$$



where $H_O = I_O \omega$ (solid body)

$H_O = r m V \sin\phi$ (point mass)

$$(H_O)_{2,\text{bullet}} + (H_O)_{2,\text{bar}} = (H_O)_{1,\text{bullet}}$$

$$(r m V \sin\phi)_{2,\text{bullet}} + \left[\left(\tfrac{1}{12}\right) m L^2 + m\left(\tfrac{L}{2}\right)^2\right]_{\text{bar}} \omega_2 = (r m V \sin\phi)_{1,\text{bullet}}$$

($\sin\phi = 1$; $r = 0.6\omega_2$)

$$(0.6\text{m})(0.030\text{kg})(0.6\omega_2)(1) + \left[\left(\tfrac{1}{12}\right)(4\text{kg})(0.75\text{m})^2 + (4\text{kg})\left(\tfrac{0.75\text{m}}{2}\right)^2\right]\omega_2 = (0.6\text{m})(0.030\text{ kg})(400\text{ m/s})(1)$$

$$\rightarrow \quad \omega_2 = +9.6\ \text{s}^{-1}$$

$$\vec{V}_G = \vec{V}_O + \vec{V}_{G/O} = (\omega_2)(r_{OG}) = (9.6)\left(\tfrac{0.75\text{m}}{2}\right) = 3.6\,\hat{i}\ \text{m/s}$$

Linear impulse provided by O_x?

$$\int_{t_1}^{t_2} O_x\, dt = G_{2,x} - G_{1,x} = \left[(4\text{kg})(3.6\text{ m/s}) + (0.030\text{kg})(9.6\,\text{s}^{-1})(0.6\text{m})\right] - (0.03\text{ kg})(400\text{ m/s})$$

$$\int_{t_1}^{t_2} O_x\, dt = \text{Linear impulse} = 2.4\ \text{N.s}\uparrow$$

Summary

- $(\vec{H}_o)_2 = (\vec{H}_o)_1 + \sum_{i=1}^{N} \int_{t_1}^{t_2} (\vec{M}_o)_i \, dt$

- $(\vec{H}_G)_2 = (\vec{H}_G)_1 + \sum_{i=1}^{N} \int_{t_1}^{t_2} (\vec{M}_G)_i \, dt$

where

- $\int_{t_1}^{t_2} \vec{M}_G dt = (\vec{M}_G)_{average} \Delta t$

angular impulse

- $\int_{t_1}^{t_2} \vec{M}_o dt = (\vec{M}_o)_{average} \Delta t$

- $H_o = I_o \omega$
- $H_G = I_G \omega$

Next Lecture

- Vibrations !

- Read Meriam 8/1 , 8/2